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The influence of a third diffusing component upon the onset of convection

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The small amplitude stability analysis for the onset of double-diffusive convection when the density gradient is gravitationally stable is extended to include a third diffusing component. Special attention is given to systems with $\kappa_1 \gg \kappa_2$, κ_3 and $Pr \gg \kappa_2/\kappa_1$, where κ_i is the molecular diffusivity of the ith component and Pr is the Prandtl number based on the largest of the κ_i. It is found that the boundary for the onset of overstability is approximated by two straight lines in a Rayleigh number plane. Small concentrations of a third property with a smaller diffusivity can have a significant effect upon the nature of diffusive instabilities, the magnitude of this effect being proportional to κ_1/κ_i. Oscillatory and direct 'salt-finger' modes are found to be simultaneously unstable under a wide range of conditions when the density gradients due to the components with the greatest and smallest diffusivities are of the same sign.

1. Introduction

Convective phenomena which are driven by the differential diffusion of two properties such as heat and salt are thought to occur in many contexts, some of which are listed by Turner (1974). To determine the conditions under which these motions will occur, the linear stability of two superposed concentration (or temperature) gradients has been studied by Stern (1960), Walin (1964), Veronis (1965), Nield (1967) and Baines & Gill (1969). Veronis (1968), Straus (1972), Huppert (1976) and Huppert & Moore (1976) have considered the stability of such a system to finite amplitude disturbances. These studies have shown that statically stable density gradients may be unstable when the density gradients due to individual components are opposed. Salt fingers can occur when the component with the smaller diffusivity is gravitationally destabilizing, while a destabilizing gradient of the faster diffusing property leads to oscillatory instability.

Only doubly diffusive convection has been considered. There are many fluid systems, however, in which more than two components are present. For example, Degens *et al.* (1973) report that the saline waters of geothermally heated Lake Kivu are strongly stratified by temperature and a salinity which is the sum of comparable concentrations of many salts, while the oceans contain many salts in concentrations less than a few per cent of the sodium chloride concentration. In laboratory experiments on double-diffusive convection, dyes or small temperature anomalies introduce a third property which affects the density of the fluid. The 'diffusive overturning' described by McIntyre (1970), which is suggested to occur in Gulf Stream eddies (Lambert 1974), also involves three components, angular velocity, temperature and salinity,

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and more complicated systems can be found in magmas and molten metals (Jakeman & Hurle 1972). Results of a small amplitude stability analysis for linear concentration gradients of three components are presented here.

2. Linearized stability analysis

Consider a fluid containing three diffusing properties and let

$$\kappa_1 > \kappa_2 > \kappa_3, \tag{2.1}$$

where κ_i is the coefficient of molecular diffusion for the ith component; let C_i be the corresponding concentration. In the Boussinesq approximation the equation of state is

$$\rho = \rho_m[1+\sum_i \beta_i C_i], \tag{2.2}$$

where ρ_m is the mean density of the system and the β_i are (local) constants.

The z' axis is chosen to be directed vertically upwards and the initial property gradients are assumed to be vertical and linear through a layer of fluid which is confined between two horizontal boundaries at $z' = 0$ and h. These boundaries are assumed to be dynamically free and perfectly conducting (or permeable) to all components. The concentration difference between the boundaries is ΔC_i with the sign convention that $\Delta C_i > 0$ when a component is 'destabilizing'. As in the two-component problem (see Baines & Gill 1969), the non-dimensional parameters are defined by scaling with h and κ_1. We then have Rayleigh numbers, R_i, diffusivity ratios τ_i and a Prandtl number Pr given by

$$R_i = h^3 g\beta_i \Delta C_i/\nu\kappa_1, \quad Pr = \nu/\kappa_1, \quad \tau_i = \kappa_i/\kappa_1 \quad (i = 1,2,3). \tag{2.3}$$

Considering only two-dimensional motions with velocity $\mathbf{q} = (u,\ 0,\ w)$, the linearized perturbation equations of vorticity and conservation of each component become

$$\left.\begin{aligned} \left(\frac{1}{Pr}\frac{\partial}{\partial t} - \nabla^2\right)\nabla^2\psi &= \sum_{i=1}^{3}\frac{\partial\theta_i}{\partial x}, \\ (\partial/\partial t - \tau_i\nabla^2)\,\theta_i &= R_i\,\partial\psi/\partial x \quad (i = 1,2,3), \end{aligned}\right\} \tag{2.4}$$

where the θ_i are non-dimensional concentrations and ψ is a two-dimensional stream function. The boundary conditions are

$$\theta_1 = \theta_2 = \theta_3 = \psi = \partial^2\psi/\partial z^2 = 0 \quad \text{at} \quad z = 0, 1.$$

Substitution of normal modes with exponential growth rate σ, horizontal wavenumber $\pi\alpha$ and vertical wavenumber π into (2.4) yields the characteristic equation

$$\begin{aligned} \frac{\sigma'^4}{Pr} + \frac{\sigma'^3}{Pr}[Pr+1+\tau_2+\tau_3] + \sigma'^2\left[1+\tau_2+\tau_3+\frac{\tau_2}{Pr}+\frac{\tau_3}{Pr}+\frac{\tau_2\tau_3}{Pr} - R_1' - R_2' - R_3'\right] \\ + \sigma'\left[\tau_2+\tau_3+\tau_2\tau_3+\frac{\tau_2\tau_3}{Pr} - (\tau_2+\tau_3)R_1' - (1+\tau_3)R_2' - (1+\tau_2)R_3'\right] \\ + \tau_2\tau_3[1-R_1'-R_2'/\tau_2-R_3'/\tau_3] = 0, \end{aligned} \tag{2.5}$$

with $\sigma' = \sigma/k^2, R_i' = (\pi^2\alpha^2/k^6)R_i$ and $k^2 = \pi^2(\alpha^2+1)$. Because (2.4) involves four rather than three time derivatives, (2.5) is of fourth order in the growth rate σ, in contrast to the cubic relation familiar from the two-component analysis. For the most unstable mode at marginal stability

$$\alpha^2 = \tfrac{1}{2}, \quad R_i' = (4/27\pi^4)\,R_i. \tag{2.6}$$

Relations (2.5) and (2.6) imply that for the most unstable mode σ is real and positive in the half space

$$\sum_{i=1}^{3} R_i \tau_i^{-1} \geqslant \frac{27\pi^4}{4}, \tag{2.7}$$

so that instability is monotonic there.† Inequality (2.7) is the generalization of the Rayleigh number criterion $R_1 \geqslant \frac{27}{4}\pi^4$ for convection involving a single diffusing component. The plane surface of marginal stability on which equality holds in (2.7) will be denoted by $\mathscr{P}$.

For the case of a marginally stable oscillatory mode we substitute $\sigma = i\sigma_i$ into (2.5) and require σ_i to be real. This yields a hyperboloidal surface $\mathscr{H}$ in R_i space, whose analytical derivation is given in the appendix and which is described in § 3.

The plane boundary $\mathscr{P}$ and the surface $\mathscr{H}$ alone still do not determine fully the conditions under which direct or oscillatory instability may occur, as there remains the possibility that complex roots of the quartic characteristic equation (2.5) may become real roots without their parts passing through zero. One such transition has been considered in the two-component case by Baines & Gill; however, two transitions occur for the quartic equation (2.5) discussed here. Numerical computations which are not presented here show that both the transition from no real to two real roots and the transition from two real to four real roots can influence the nature of the mode of instability. In § 3 possible positions of the boundaries $\mathscr{P}$ and $\mathscr{H}$ are illustrated for two sets of molecular properties.

3. Examples of stability boundaries

3.1. *A case where $\tau_2, \tau_3 \lesssim 1$*

Figure 1 (*a*) shows the relevant portions of the intersections of $\mathscr{P}$ and of $\mathscr{H}$ with the plane $R_3' = -5$ [see (2.6)]; 'relevant portions' means those which describe a change in the mode of instability for the most unstable mode. The numerical parameter values used are $\kappa_1 = 1{\cdot}6 \times 10^{-5}$ cm^2 s^{-1}, $\kappa_2 = 1{\cdot}3 \times 10^{-5}$ cm^2 s^{-1}, $\kappa_3 = 0{\cdot}45 \times 10^{-5}$ cm^2 s^{-1} and $Pr = \nu/\kappa_1 = 625$, corresponding to an aqueous solution of KCl, NaCl and sucrose. The diffusivity ratios are not far from unity, as is typical of molecular species in molten metals and magmas as well as of laboratory models using salt–sugar solutions. Figures 1 (*b*) and (*c*) show the same for the planes $R_3' = 0$ and $R_3' = 5$. The plane $\mathscr{S}$ defined by

$$\sum_i R_i = 0$$

and on which $\partial\rho/\partial z = 0$ is also shown.

† On the plane of marginal stability one root of (2.5) and (2.6) is $\sigma = 0$. Although this is not always the root of maximum growth rate, numerical computations of the nature of the four roots show that 'exchange of stabilities' does occur when equality holds in (2.7) if the density gradient is gravitationally stable.

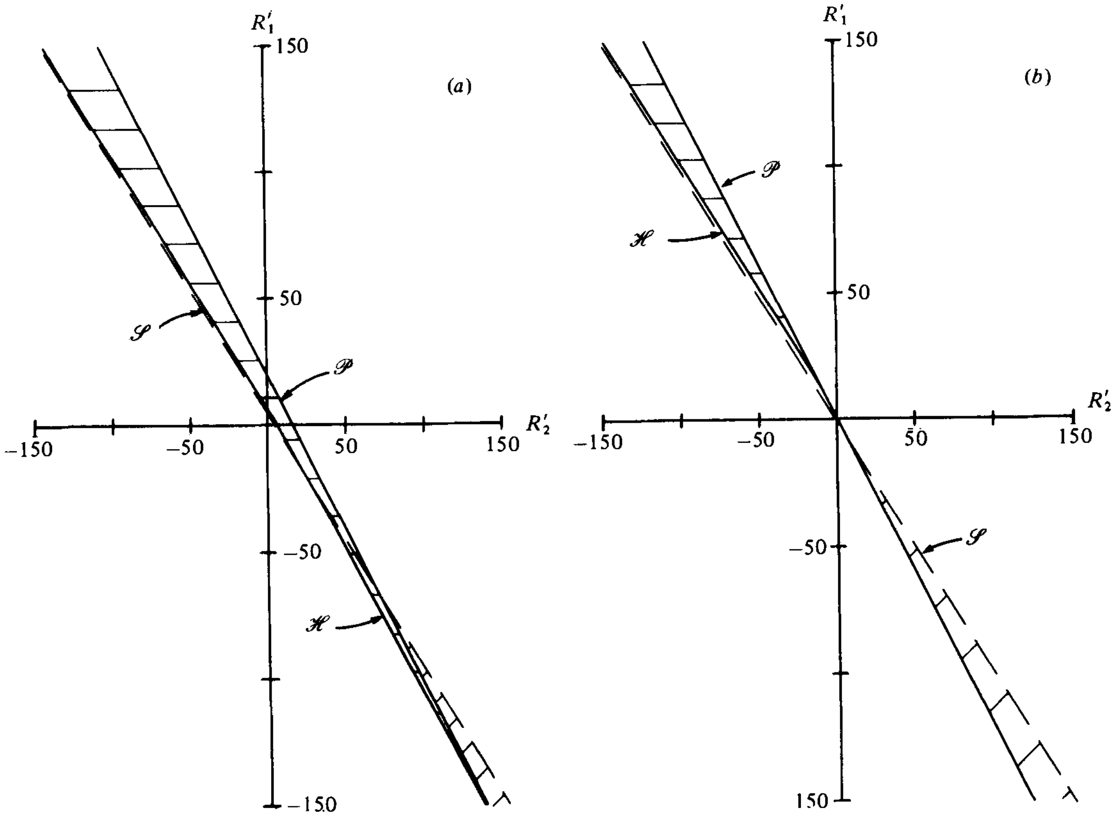

FIGURE 1(*a*,*b*). For legend see facing page.

The asymptotes of the hyperbola are almost parallel, the slope of each being independent of the value of R_3'. The upper asymptote has slope $-(Pr+\tau_2)/(Pr+1)$. Salt fingers and overstable modes can occur (as the most unstable mode) at adjacent conditions when the fastest and slowest diffusing components are destabilizing (second quadrant of figure 1 (*c*) when R_3' is sufficiently laige) and more extensively when these components are stabilizing (fourth quadrant of figure 1 *a*).

3.2. *A case where* $\tau_2, \tau_3 \ll 1$

The discussion of a three-component system may be simplified when τ_2, $\tau_3 \rightarrow 0$ and $Pr \gg \tau_2$ since $\mathscr{H}$ approaches its asymptotes in this limit and the asymptotes themselves degenerate into a pair of planes:

$$R_1 + \frac{Pr}{(Pr+1)}(R_2+R_3) = \tfrac{27}{4}\pi^4, \tag{3.1a}$$

$$R_1 + (\tau_2+\tau_3)^{-1}(R_2+R_3) = \tfrac{27}{4}\pi^4. \tag{3.1b}$$

When $R_3 = 0$, relation (3.1 *a*) reduces to the condition for marginal stability of oscillatory modes of two-component systems.

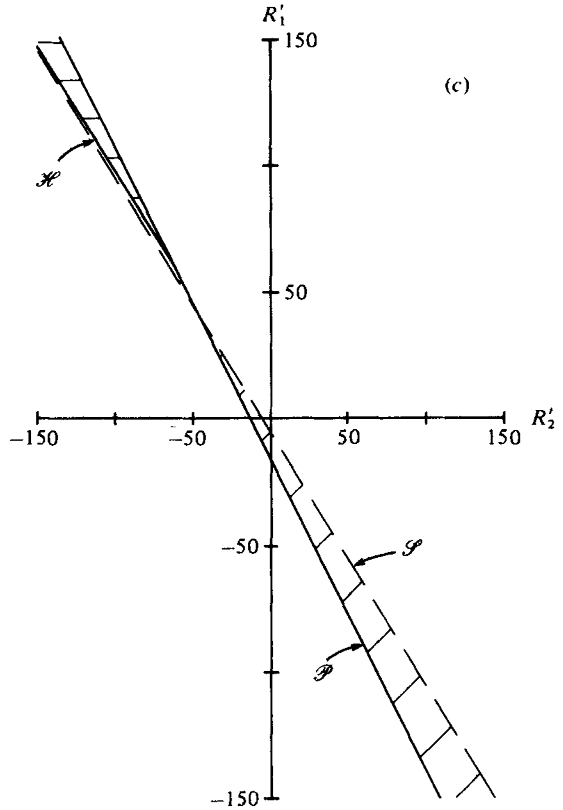

FIGURE 1. Stability boundaries for the most unstable mode when $\tau_2 = 0{\cdot}81$, $\tau_3 = 0{\cdot}28$, $\tau_* = 0{\cdot}35$ and $Pr = 625$ at three values of R_3': (*a*) $R_3' = -5$; (*b*) $R_3' = 0$; (*c*) $R_3' = 5$. The co-ordinates are normalized with respect to the critical Rayleigh number $\frac{27}{4}\pi^4$ and the lines are explained in the text. Horizontally hatched regions give overstable modes; oblique hatching shows conditions unstable to salt fingers.

Here we use as an example the aqueous system heat–KCl–sucrose ($i = 1, 2, 3$ respectively), for which

$$\kappa_1 = 1{\cdot}4\times10^{-3}\ \mathrm{cm^2\,s^{-1}},\quad \kappa_2 = 1{\cdot}6\times10^{-5}\ \mathrm{cm^2\,s^{-1}},$$

$$\kappa_3 = 0{\cdot}45\times10^{-5}\ \mathrm{cm^2\,s^{-1}}\quad \text{and}\quad Pr = 7.$$

Figures 2(*a*), (*b*) and (*c*) show the relevant portions of the intersections of $\mathscr{P}$, $\mathscr{H}$ and $\mathscr{S}$ with the three planes $R_3' = -0{\cdot}96$, $R_3' = 0$ and $R_3' = 0{\cdot}96$, respectively. It can be seen that $\mathscr{H}$ is described very closely by (3.1).

To show the three-dimensional geometry more clearly the intersections of the surfaces $\mathscr{P}$, $\mathscr{H}$ and $\mathscr{S}$ with each other, as functions of R_3', will also be defined and are shown as broken lines in figure 2. The two asymptotes (3.1) intersect at the point A where $R_1' = 1$ and $R_2' = -R_3'$. This point lies to the left of $\mathscr{P}$ when $R_3' < 0$ (see figure 2*a*). Then the lower asymptote (3.1*b*) of $\mathscr{H}$ and the plane $\mathscr{P}$ converge slowly to intersect at P, a point where $R_1' = 1-|R_3'|(1-\tau_*)/(\tau_*\tau_3)$ and $R_2' = |R_3'|\tau_*^{-2}$. Here $\tau_* = \tau_3/\tau_2 = \kappa_3/\kappa_2$ and the almost vertical broken line in the fourth quadrant of figure 2(*a*) is the locus of P as R_3' is varied. The point P lies well within the region of static stability

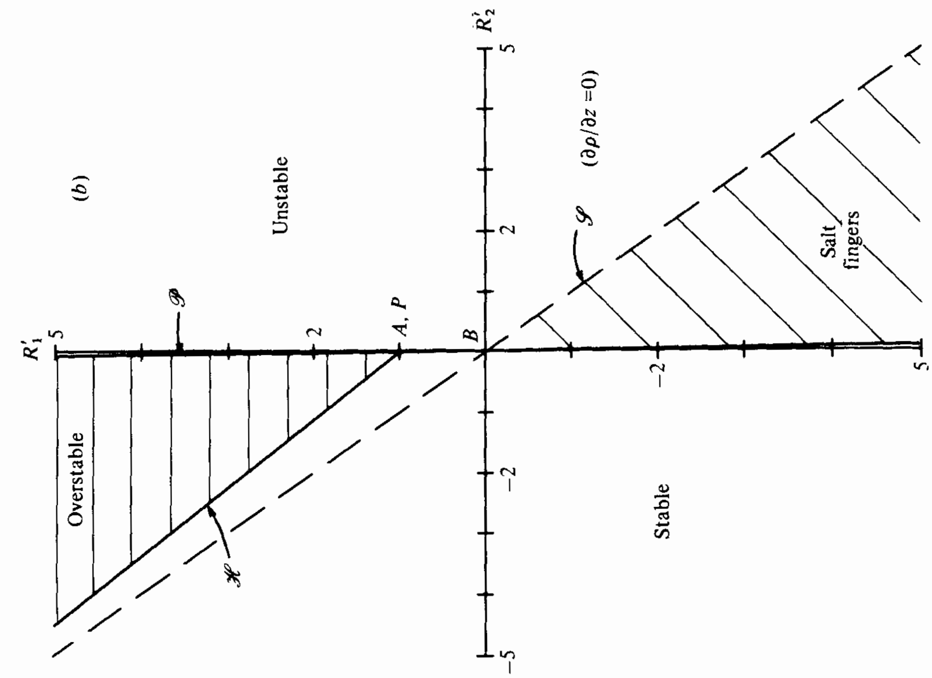

FIGURE 2(*b*). For legend see facing page.

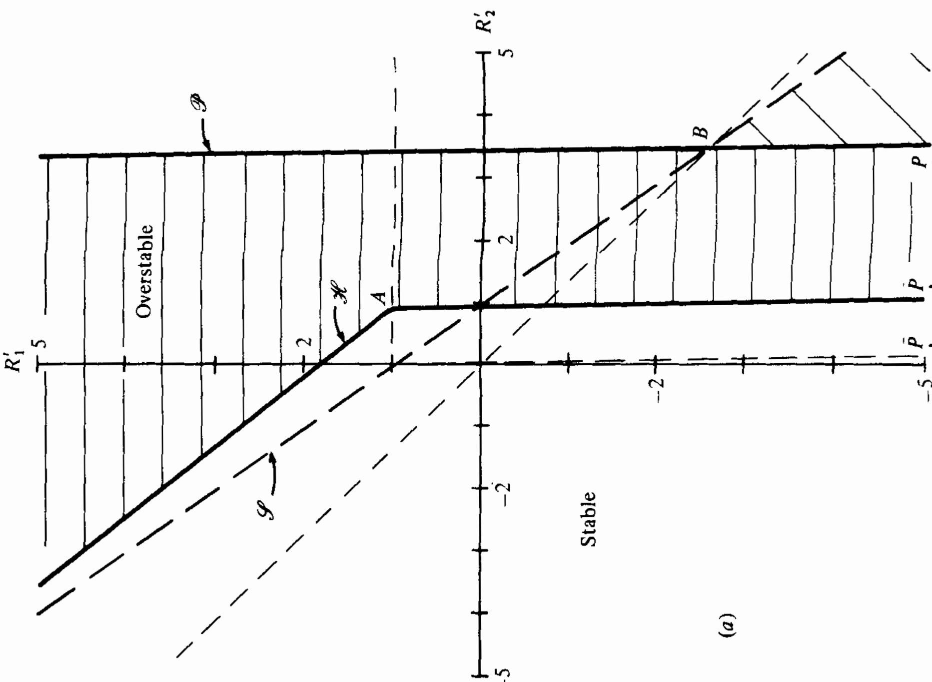

FIGURE 2(*a*). For legend see facing page.

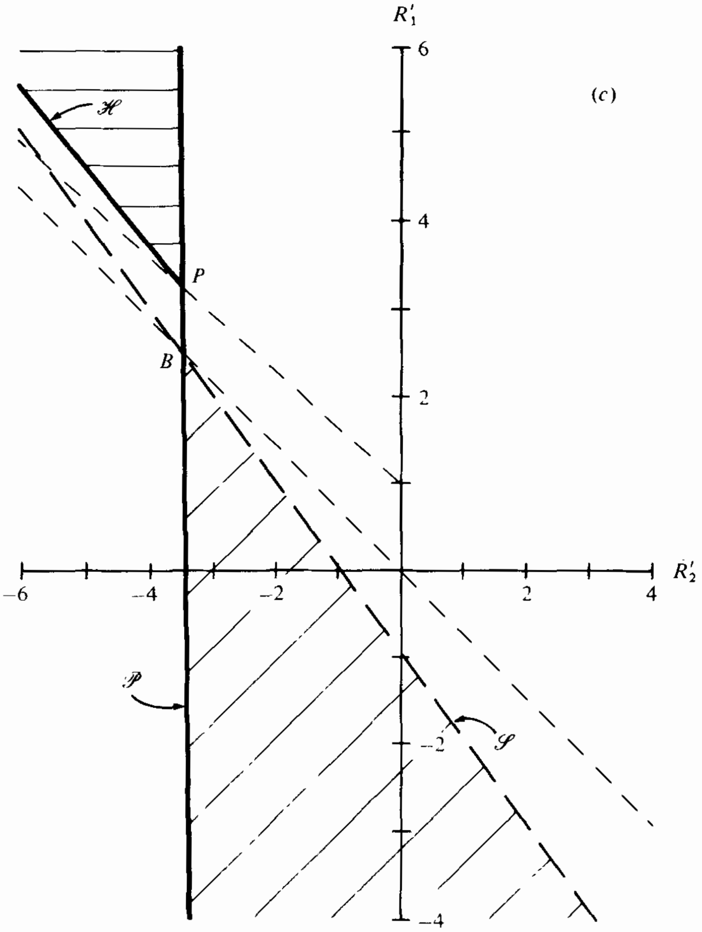

FIGURE 2. Stability boundaries for the most unstable mode when $\tau_2 = 0{\cdot}011$, $\tau_3 = 0{\cdot}003$, $\tau_* = 0{\cdot}28$ and $Pr = 7$ at three values of R_3': (*a*) $R_3' = -0{\cdot}96$; (*b*) $R_3' = 0$; (*c*) $R_3' = 0{\cdot}96$. Hatching and heavy lines have the same meaning as in figure 1 and the points A, B and P move along the fine broken lines when R_3' is varied.

(provided that $R_3' < 0$), so that a stabilizing gradient of the component with smallest diffusivity causes a large extension of the range of values of R_1' and R_2' at which over-stable modes occur. Further, P falls well below B, the intersection of $\mathscr{P}$ and $\mathscr{S}$.

For $R_3' > 0$ (see figure 2*c*) the point A lies to the right of $\mathscr{P}$, so that only one asymptote, (3.1*a*), is drawn. The intersection P now lies on the line $R_1' + Pr\,(1-\tau_*)\,R_2'/(Pr+1) = 1$ in the second quadrant of the R_2', R_1' plane. There are two points of interest: it may be shown that P again falls below B when $R_3' > (Pr+1)\,\tau_*/(1-\tau_*)$ ($\approx 3{\cdot}1$ in the present example); also, more obviously, a destabilizing gradient of component 2 ($R_2' > 0$) is no longer a necessary condition for the growth of salt fingers.

Through the use of (2.6) in the above discussion, we have considered only the nature of the most unstable mode near marginal stability. As shown in § 4, the stability analysis yields further information about the physical behaviour of multi-component systems.

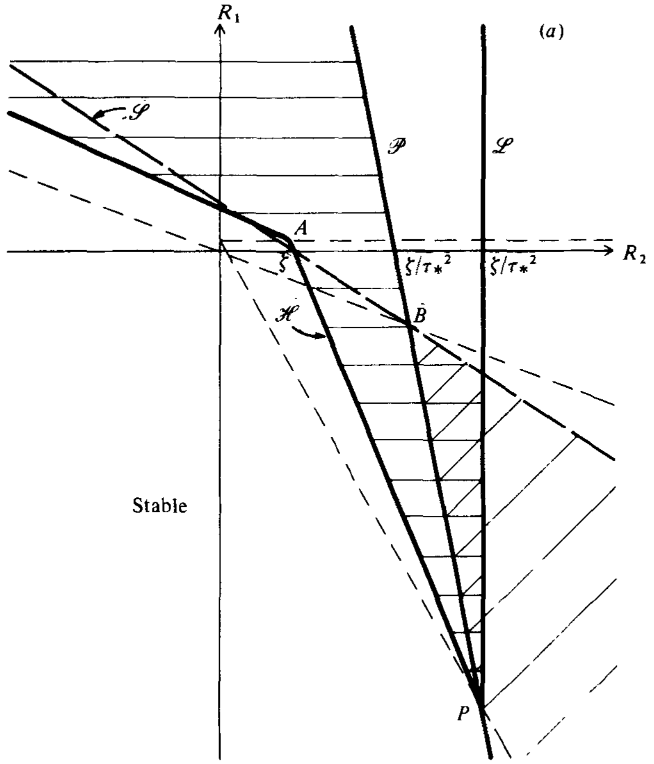

FIGURE 3(a). For legend see facing page.

4. Physical behaviour of three-component systems

We no longer restrict attention to the most unstable mode and allow α to vary. The portion of the plane $\mathscr{P}$ above its intersection with $\mathscr{H}$ (at P in figure 2) must now divide conditions to the left at which only oscillatory disturbances are unstable from conditions to the right at which both overstable and monotonically growing modes are possible. This behaviour is illustrated schematically on the R_2, R_1 plane in figure 3(a) for $R_3 = -\zeta < 0$ and in figure 3(b) for $R_3 = \eta > 0$. As R_3 is varied the intersections A, B and P move along the fine broken lines; the values of R_2 at intersections of $\mathscr{S}$, $\mathscr{P}$ and $\mathscr{L}$ with the R_2 axis are marked.

In these diagrams another plane surface $\mathscr{L}$ has been defined, to the right of which no wavelengths are overstable and only monotonic instability is possible. This plane $\mathscr{L}$ is the locus of the intersections of $\mathscr{H}$ and $\mathscr{P}$ as the term $\pi^2\alpha^2/k^6$ is allowed to vary in (2.5). In the limit τ_2, $\tau_3 \rightarrow 0$, (2.5), (2.7) and (3.1) imply that $\mathscr{L}$ is vertical, while for the two-component case and arbitrary molecular properties Baines & Gill (1969) find the appropriate line to be (in the notation of this paper)

$$R_1 = \frac{-(Pr+\tau_2)}{(Pr+1)\tau_2^2} R_2.$$

To explore the behaviour further, consider a system which lies in the fourth quadrant of figure 3(a) and which always has $\partial\rho/\partial z < 0$. Then the components with the greatest and smallest diffusivities (κ_1 and κ_3) both contribute negative density gradients

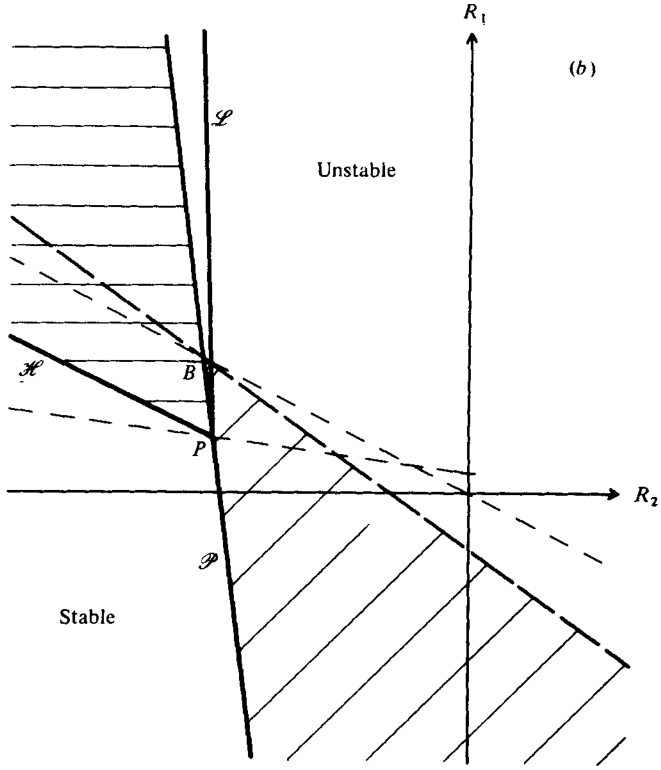

FIGURE 3. Schematic three-component stability bounds (heavy lines) for (a) $R_3 = -\zeta < 0$, stabilizing, and (b) $R_3 = \eta > 0$, destabilizing. Values of R_2 at intersections with $R_1 = 0$ are marked. Both oscillatory and salt-finger modes are unstable in the double-hatched region.

opposing that due to component 2. When R_2 is sufficiently small the system is stable. However, if R_2 is increased until the (R_1, R_2, R_3) co-ordinates cross $\mathscr{H}$ then some wavelengths become overstable, beginning at $\alpha = 2^{-\frac{1}{2}}$, the mode which represents a balance between more efficient diffusive transport and viscous damping. The oscillatory modes can transport more (in density terms) of component 2 by simple molecular diffusion than of components 1 and 3 combined, even though such disturbances transport components with greater diffusivity most rapidly. If R_2 is more destabilizing still, so that the conditions are just to the right of $\mathscr{P}$, the mode with wavenumber $\alpha = 2^{-\frac{1}{2}}$ grows monotonically while other modes remain overstable. R_2 is now large enough to overcome the stable stratification as well as viscosity and at larger values (to the right of $\mathscr{L}$) all unstable modes are direct.

Overstable modes are not possible when a sufficiently large 'stabilizing' gradient of component 1 relative to $|R_3|$ tends to cause a net upward diffusive density flux through oscillatory disturbances. However, direct modes can decrease the system's potential energy by preferential transport of components with lower diffusivity. On the other hand, direct instability cannot occur with $\partial\rho/\partial z < 0$ if $|R_1|$ is sufficiently small compared with $|R_3|$ since component 3 would then cause a net upward density flux. The subsequent finite amplitude motions have not been considered in the above discussion but, where both forms of instability are possible, they will depend upon the relative growth rates of the two forms, and the fastest growing mode (assuming no interactions) may be calculated from the dispersion relation (2.5) at given Rayleigh number co-ordinates.

5. Conclusions

Two results of the analysis for triple-diffusive instability (assuming $Pr \gg \kappa_2/\kappa_1$) are (i) that marginal stability of oscillatory modes occurs on a hyperboloid in Rayleigh number space but the surface is very closely approximated by its planar asymptotes for any diffusivity ratios, and (ii) that, when the density gradients due to the components with the greatest and smallest diffusivities are of the same sign, salt-finger and oscillatory modes may be simultaneously unstable over a wide range of conditions.

One of the three components may be neglected under conditions suggested by the stability boundaries. The influence of a component upon marginally stable oscillatory modes is proportional to $|R_i|$(or $\beta_i \Delta C_i$), while the influence of any component upon the occurrence of salt fingers is proportional to $|\beta_i \Delta C_i| \kappa_i^{-1}$. Thus small concentrations of slowly diffusing properties can be important. It might be noted here that the molecular diffusivity of suspended matter is many orders of magnitude less than that of salt. For cases in which κ_2, $\kappa_3 \ll \kappa_1$, the condition (3.1) for neutral stability of oscillatory modes can be written in terms of a total (salinity) Rayleigh number R_s, where $R_s = R_2 + R_3$. Further, the onset of overstability is independent of the individual slower diffusing species present whenever temperature is sufficiently destabilizing to produce (thermal) convection if it alone was present. Once convection is proceeding, however, the flux of each component down its concentration gradient may be influenced by the individual molecular diffusion coefficients. Further work (such as the experiments by Turner, Shirtcliffe & Brewer 1970) is needed to establish the influence of a third component upon the fluxes through the resulting large amplitude convection, in particular, through 'salt-finger' and 'diffusive' interfaces.

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Appendix

We substitute $\sigma = i\sigma_i$ into (2.5). Collecting real parts gives

$$\sigma_i'^4 Pr^{-1} - \sigma_i'^2(D - R_1' - R_2' - R_3') + \tau_2\tau_3(1 - R_1' - R_2'\tau_2^{-1} - R_3'\tau_3^{-1}) = 0, \qquad (\text{A } 1)$$

and from the imaginary parts

$$\sigma_i'^2 = (Pr/B)[A - (\tau_2+\tau_3) R_1' - (1+\tau_3) R_2' - (1+\tau_2) R_3'], \qquad (\text{A } 2)$$

where

$$A = \tau_2 + \tau_3 + \tau_2\tau_3(1 + Pr^{-1}),$$
$$B = Pr + 1 + \tau_2 + \tau_3,$$
$$D = 1 + \tau_2 + \tau_3 + Pr^{-1}(\tau_2 + \tau_3 + \tau_2\tau_3).$$

Together, (2.6), (A 1) and (A 2) describe the curve

$$a_1 R_1^2 + a_2 R_2^2 + a_3 R_3^2 + a_4 R_1 R_2 + a_5 R_1 R_3 + a_6 R_2 R_3 + \tfrac{27}{4}\pi^4[a_7 R_1 + a_8 R_2 + a_9 R_3] + (\tfrac{27}{4}\pi^4)^2 a_0 = 0, \qquad (\text{A } 3)$$

where the coefficients are

$$
\begin{aligned}
a_1 &= -(Pr+1)(\tau_2+\tau_3),\\
a_2 &= -(Pr+\tau_2)(1+\tau_3),\\
a_3 &= -(Pr+\tau_3)(1+\tau_2),\\
a_4 &= 2(1+\tau_3)(\tau_2+\tau_3)-B(1+\tau_2+2\tau_3),\\
a_5 &= 2(1+\tau_2)(\tau_2+\tau_3)-B(1+2\tau_2+\tau_3),\\
a_6 &= 2(1+\tau_2)(1+\tau_3)-B(2+\tau_2+\tau_3),\\
a_7 &= (BD-2A)(\tau_2+\tau_3)+AB-\tau_2\tau_3 B^2Pr^{-1},\\
a_8 &= (BD-2A)(1+\tau_3)+AB-\tau_3 B^2Pr^{-1},\\
a_9 &= (BD-2A)(1+\tau_2)+AB-\tau_2 B^2Pr^{-1},\\
a_0 &= A^2-ABD+\tau_2\tau_3 B^2Pr^{-1}.
\end{aligned}
$$

We also require $\sigma_i'^2 > 0$ in (A 1) and (A 2). Thus the inequalities

$$(\tau_2+\tau_3)R_1+(1+\tau_3)R_2+(1+\tau_2)R_3 \leqslant \tfrac{27}{4}\pi^4 A$$

and

$$\tfrac{27}{4}\pi^4 D-R_1-R_2-R_3 \pm G^{\frac{1}{2}} \geqslant 0 \tag{A 4}$$

must be satisfied on (A 3), where

$$
\begin{aligned}
G = {} & R_1^2+R_2^2+R_3^2+2(R_1R_2+R_1R_3+R_2R_3)\\
& +\tfrac{27}{4}\pi^4[4\tau_2\tau_3 Pr^{-1}(R_1+R_2\tau_2^{-1}+R_3\tau_3^{-1})-2D(R_1+R_2+R_3)]\\
& +(\tfrac{27}{4}\pi^4)^2(D^2-4\tau_2\tau_3 Pr^{-1}).
\end{aligned}
$$

Whenever $0<\tau_2$ and $\tau_3<1$, (A 3) is an hyperboloid of which inequalities (A 4) select the appropriate branch: the conditions at which the most unstable oscillatory mode is marginally stable. It becomes a paraboloid when $\tau_2, \tau_3 = 1$, i.e. when only one component is present, and its intersection with any plane parallel to the $R_1 = 0$ plane becomes parabolic when $\tau_2, \tau_3 = 0$.

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